Using Self-Driving Cars to Increase Highway Traffic Flow: The Results of a Cellular Automata Simulation

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A cellular automata simulation of highway traffic flow was run to study the effects on large-scale traffic flow of self-driving cars mixed in with human drivers. When 80 percent of cars on the highway were self-driving, peak flow was approximately 24 percent higher than with only human drivers, and seriously congested flow did not occur until a density twice as high. Optimal behavior for self-driving cars was also studied. For the range of behaviors tested, optimal behavior included leaving a small gap when following cars close ahead, and checking behind when changing lanes to avoid cutting other cars off. These behaviors will need to be considered in addition to basic behaviors like avoiding crashes and obeying the speed limit when designing self-driving cars to be used on public roads.
I. Why Study Traffic Flow?

Highway traffic flow is an incredibly important problem to study in order to increase efficiency and reduce pollution. Increasing traffic flow allows cars to operate closer to optimal speed, thus reducing natural gas use and greenhouse gas emissions. Increased traffic flow also allows cars to reach their destinations sooner, which means less total running time for car engines. In 2012, fuel burned in transportation accounted for 28 percent of United States carbon dioxide emissions. Of this 28 percent, passenger cars and light-duty trucks accounted for more than half of emissions (EPA 2014), so increasing highway efficiency will have a significant impact on fuel consumption and greenhouse gas emissions.

Highway traffic flow also impacts safety. In optimal traffic flow, drivers change lanes and brake less often, making accidents much less likely. In 2012, there were over 5.5 million motor vehicle crashes in the United States, which killed over 33,000 people (an average of 92 per day) and injured over 2 million more (NHTSA 2014). Changing traffic flow to make accidents less likely could save thousands of lives each year.

Human behavior is notoriously difficult to change, so until recently there was little anyone could do to significantly increase highway traffic flow other than building wider roads. Self-driving and semi-autonomous cars provide an unprecedented opportunity to actually increase traffic flow, decrease natural gas use, and increase safety. This paper will first give a brief overview of highway traffic flow research and self-driving car research to date, and then present the results of a new simulation involving self-driving cars mixed in with human drivers.

The end goal of this research is to understand how to design connected cars, self-driving cars, and cars with simpler technology like adaptive cruise control to increase traffic flow. This will be an important problem to understand and solve in the future as these cars make up a significant fraction of all cars on the highway. But more importantly, these new types of cars give us the first opportunity ever to change traffic flow on a large scale simply by changing the microscopic behavior of cars.
Although human behavior is hard to modify, changing the behavior of computerized cars is as easy as downloading a software update — something that could even be done on the fly, for example, to alert cars about new roads. With a solid understanding of highway traffic flow, it will be possible to program computerized cars to significantly increase flow without having to modify human behavior much at all. Previous studies have found that the nature of traffic flow can change significantly when as little as ten percent of cars on the road exhibit different behavior (Li et al. 2006), so it may not be long before programming traffic flow is a reality.

Nobody to date has modeled highway traffic flow perfectly, but some studies have reproduced many of the key features of real traffic flow. A variety of models have been tried by various researchers, from physics-based models to rule-based simulation models (Schadschneider 2002). Some physics-based models treat traffic as a compressible fluid and model it using fluid dynamics equations, while others treat it using gas laws and thermodynamic equations. Rule-based models offer a completely different approach. Instead of trying to solve equations, they start by modeling driver behavior and then simulating traffic using computers. Although these models are not structured in the same way as many models in physics, they lead to similar results in simulations and have many other advantages.

The most fundamental feature of highway traffic flow is the flow rate. The flow rate depends on many variables such as speed limit, number of lanes, weather conditions, driver behavior, vehicle characteristics, and vehicle density. For most models, the goal is to accurately predict the flow as a function of density using realistic values for the other conditions. Once the accuracy of the model is established using common values for the basic parameters, the model can be used to look at microscopic and macroscopic traffic behavior, phase transitions, traffic jam formation and resolution, and other interesting phenomena. As recently as the 1990's, simulation speed was a critical concern when studying traffic flow (Nagel and Schreckenberg 1992). Since computers have only recently become powerful enough to run large simulations of traffic flow, many possibilities for modeling have yet to be explored. In addition, some models that have been explored in the past can be improved by more realistically simulating driver behavior.
II. The State of Self-Driving and Semi-Autonomous Cars

Until recently, successful self-driving car demonstrations were limited to research groups, most notably the groups participating in the DARPA Challenge (DARPA 2014). In the DARPA Grand Challenge, and related challenges also associated with DARPA, research groups were challenged to design vehicles that could navigate a 100-plus mile course as fast as possible without crashing. The first iteration of the contest featured a challenging desert course, with obstacles not seen on normal roads. A later challenge was done in an urban environment that required some interaction with traffic, but only for part of the challenge.

Within the last few years, however, research has progressed significantly: there are now cars that can drive autonomously on public roads without any human interaction whatsoever. The most prominent effort comes from Google, whose cars have driven hundreds of thousands of miles with only occasional human intervention (Fisher 2013). Unlike previous autonomous car projects, which were designed more for military applications, current projects like Google’s are aimed at the general public, with the eventual goal of replacing many human drivers on public roads. These cars rely on LIDAR (laser radar) and other sensors to create high-resolution three-dimensional images of the surrounding environment. They process the image data in real time and react practically instantaneously — much faster than human drivers.

In addition to Google’s efforts, luxury car companies have been begun to implement limited features of self-driving cars, such as adaptive cruise control, lane departure warning systems, and automated parking. These efforts bridge the gap between human drivers and self-driving cars, and insofar as they affect driver behavior, could have similar benefits to self-driving cars on a large scale. But car companies are not stopping halfway. Just this year, Audi and BMW demonstrated their own self-driving cars (Kelly 2014). Although the cars are not yet as advanced as Google’s, they are making progress. BMW’s autonomous control system, for example, is already advanced enough to control all of the car’s driving systems, and can enter and exit a controlled skid entirely on its own.
III. The Fundamental Diagram

The graph of the relationship between density and flow is called the fundamental diagram. It looks very similar to a phase diagram typical of statistical mechanics. Figure 1, below, shows a generalized fundamental diagram. The diagram is a plot of flow rate \( J \), measured as the number of cars that pass a fixed point over a time interval) as a function of density \( \rho \). The density can either be measured globally as the fraction of space on a road occupied by cars, or locally as the fraction of time that a fixed point on the road is occupied by cars.

![Figure 1: A generic fundamental diagram](image)

In the free flow region, an increase in density leads to a proportional increase in flow. In congested flow, increases in density decrease flow. There is much more variation in flow for intermediate and high densities. The density at the boundary between free flow and congested flow is called the critical density.

In realistic fundamental diagrams, there are at least two phases of traffic flow: free flow and congested flow. In free flow, cars move close to the speed limit, and an increase in density leads to an increase in flow (indicated by the positive slope in the fundamental diagram). In the congested phase, cars do not generally move close to the speed limit, and an increase in density leads to a decrease in flow. This negative definition of congested flow has led to debates about whether congested flow really
encompasses multiple distinct types of flow (Chowdhury et al. 2002), but in general, it just means the flow is jammed.

The large size of the gray region in Figure 1 indicates how much congested flow can vary. This variation depends on many factors — weather, time of day, road conditions, etc. — many of which are mediated through driver behavior. By changing driver behavior (e.g. through driver education) or by introducing self-driving and semi-autonomous cars, it may be possible to significantly increase flow at intermediate and high densities.

Distinct flow phases are determined partly by looking for changes in the slope of the fundamental diagram. At the critical density, flow decreases sharply because the traffic undergoes a phase change. The qualitative nature of this phase change is easy to understand from experience. In free flow, most cars move close to the speed limit and do not change their speeds significantly (Figure 2). In congested flow, most cars cannot move close to the speed limit, and in many cases change their speeds frequently, as in stop-and-go traffic. Since the microscopic nature of the flow is different in congested traffic, the relationship between flow and density is different. Figure 2, below, shows a qualitative plot of this behavior.

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**Figure 2:** Variance in car speeds at different densities

In free flow, most cars move close to the speed limit. Past the critical density, most cars cannot maintain speeds near the speed limit. At extremely high densities, all cars travel slowly, and generally exhibit stop-and-go behavior.
IV. A Survey of Available Models

In his 2002 paper, Andreas Schadschneider surveys a wide variety of models that have been used recently. Although the paper is now over a decade old, the types of models used then are the same as now, just with slightly less complexity. The two macroscopic physics-based models covered are hydrodynamic models and gas-kinetic models. Both of these models have been able to produce accurate results, but because they are macroscopic, it is hard to adjust the models to account for specific changes in driver behavior (Schadschneider 2002).

Two microscopic physics-based models are car following models and optimal velocity models. Car following models use methods of Newtonian mechanics to determine car motion (Schadschneider 2002). Specifically, these models have cars react to stimuli such as the speed of the car ahead and the distance to cars ahead and behind; these reactions can be formulated similar to kinematic equations of classical mechanics. Although this approach seems attractive, Schadschneider finds serious problems with it in low-density situations because the structure of the model depends on interactions between cars, which only happen over short distances.

Optimal velocity models are similar to car following models, but they fix the problems in the low-density limit by giving cars a target velocity in the absence of interactions with other cars. This fixes the low-density problem of car following models, but Schadschneider finds that the optimal velocity model is not collision free in its most basic form. Although collisions happen in real traffic, it is hard to match the rate of collisions in a model with the rate in the real world when that rate is a direct consequence of the basic structure of the model.

The final class of models considered by Schadschneider in his comparison is cellular automata. In these models, the highway is split into discrete cells, each of which can be left empty or occupied by a single car. The positions of the cars are updated at each time step in the simulation according to a set of rules. This simple structure makes it easy to simulate specific driver behavior. With the increased speed of modern computers, it is also practical to run large simulations of cellular automata models.
Schadschneider and many others have seen results from cellular automata models that match real-world observations of traffic flow on highways (e.g. Nagel and Schreckenberg 1992), so given the two benefits just mentioned, cellular automata models are currently one of the most attractive options for understanding and modeling traffic flow. One downside is that analyzing these types of models mathematically becomes much harder once the model becomes complex (Nagel and Schreckenberg 1992). However, with more research, it may be possible to use probabilistic methods to understand cellular automata models for traffic flow mathematically.
V. The Nagel–Schreckenberg Cellular Automata Model

One model first published in 1992 by Nagel and Schreckenberg has spawned many new versions that are remarkably good at simulating real traffic flow, while also being very easy to understand and modify. The simulation begins with a vector representing a single-lane highway. It then fills up some of the spots on the road with cars, which initially have speed 0. At each time step, it calculates new speeds for all the cars, and then updates the positions accordingly. The model divides the road into discrete chunks (so that they can be represented easily by a vector), so it is a cellular automata model.

The rules for updating the speeds ($v$) are:

1) If $v$ is less than the speed limit, it is increased by 1.
2) If the next car ahead is within $v$ cells of the current car, $v$ is reduced sufficiently so that the current car does not crash.
3) If $v$ is nonzero, it is reduced by 1 with probability $p$.

This model is a minimal model because if any of the rules is removed, it will not simulate real traffic flow. Without the first rule, cars could eventually travel at unrealistic speeds. Without the second rule, cars would always crash when approaching slower cars. Without the third rule, the model would be deterministic, and the phase change between free flow and congested flow would not fit measurements from real roads. Interestingly, acceleration (rule 1) followed by braking (rule 2), are sufficient to cause jamming, but rule 3 is necessary to produce the right kind of jamming.

In their first paper using this model, Nagel and Schreckenberg showed that the model accurately reproduces features of real traffic flow (1992). Although it was possible for them to more accurately model driver behavior, that would have caused the simulation to run more slowly. An important result of their initial paper is that traffic flow can be modeled accurately even with a simplistic model of driver behavior. Although the Nagel–Schreckenberg model is less obviously related to statistical physics and fluid dynamics than other models, it has all of the advantages of cellular automata models mentioned earlier.
The fundamental diagram produced by the original Nagel–Schreckenberg model is shown below in Figure 3, next to Nagel and Schreckenberg’s original results taken from an actual highway. These results show the same basic shape as real traffic flow but without the sharp discontinuity at the transition point to congested flow. Part of that discontinuity lies simply in the domain, so the difference between the data is not even as large as it appears on the plots — there were simply fewer measurements taken near the critical density.

Given the simplicity of the model, this accuracy is remarkable. In addition to reproducing a realistic fundamental diagram, Nagel and Schreckenberg also found that traffic jams can form simply as a result of random braking — a surprisingly simple and loose condition. Although the scales on the plots are different (since the real-world size represented by each vector position and the time step can both be arbitrary), Nagel and Schreckenberg calculated based on the flow rate that the scales actually match up. This serves as another verification of the accuracy of the model.

![Figure 3: Nagel and Schreckenberg’s original results](image)

Each dot on the left plot is an average over 100 time steps. The line is an average over all time steps. Each dot on the right plot represents a separate observation of flow.

Because the overall structure of the Nagel–Schreckenberg model does not depend on how the new velocity is computed, it is almost trivially easy to add new conditions to this update that more accurately reflect driver behavior. For example, drivers might be more likely to slow down
randomly if there are cars nearby. Some drivers may choose not to travel at the speed limit, while others may choose to travel just above it. The options here are practically limitless. The structure of cellular automata models in general is similarly easy to change. Some additions include roads with multiple lanes, vehicles that occupy two cells at a time (for example, trucks and buses), and roads with different speed limits in different locations.

Since the original paper was published, there have been dozens of modifications made to the Nagel–Schreckenberg model that take into account different driver characteristics and highway scenarios. The most basic modification is to allow for multiple lanes and hence lane-changing behavior. In addition, some researchers have studied the effects of introducing a slow-moving bus into the model, while others have studied the effects of aggressive drivers. Although these particular modifications are not the focus of the research for this paper, studying their rules for lane changing has helped to increase the accuracy of the model used here.

A 2006 paper by Li et al. is a good example of the type of results that can be found by studying modifications of the Nagel–Schreckenberg model and other cellular automata models. Li et al. studied the effects of lane-changing behavior for fast-moving vehicles on a two-lane highway. They found that for low and intermediate densities, allowing fast-moving vehicles to change lanes aggressively increased flow (Li et al. 2006). This makes sense because it allows the fast-moving vehicles to get to their destinations faster without significantly disturbing other vehicles.

However, past a certain density, they found that aggressive lane changing decreases flow. This is because past a certain density, aggressive lane changing behavior leads to turbulence — that is, greater variance in car speeds — in traffic, which decreases flow. The researchers also considered how the ratio of slow to fast cars on the highway affected flow. They found that with very few fast cars, aggressive lane changing made a positive difference regardless of density, but only slightly. This is because it takes a sufficient number of fast cars to cause enough turbulence to decrease flow.
VI. Summary of Research Methods

This paper uses a cellular automata model for highway traffic flow loosely based on Nagel and Schreckenberg’s. The model is also similar to the one used by Li et al. Their model uses two kinds of cars: fast aggressive cars and normal cars. The model for this paper allows for some proportion of cars on the road to be self-driving, with the rest of the cars set as human drivers. This is not as complicated as programming an actual car to be self-driving. Instead, it simply requires determining the behavior that such cars should exhibit, and programming the model so that the self-driving cars follow the set of rules that generates that behavior.

Rather than just studying one behavior, the effects of different self-driving car behaviors were tested for this study. The final results use the behavior that provided the largest increase in flow. All of the behaviors tested were reasonable on physical grounds, so the results of the study are realistic. For example, it would be easy to increase flow in a simulation by allowing cars to jump over other cars or drive twice the speed limit but never crash, but none of the behaviors considered were like this. This optimal behavior, and the details of the model in general, are described in the next section.

The simulation was run in Matlab R2013a (version 8.1.0.604, by Mathworks, Inc. Natick, MA). The code has the same structure as the original Nagel–Schreckenberg model, but it allows for any number of lanes, any number of cars, two types of cars, and a variety of driver behavior for each type of car.
VII. Model Description

The cellular automata model for this study is loosely based on the Nagel–Schreckenberg model, and was used to simulate highway traffic flow with two types of cars: cars driven by humans and self-driving cars. Human-driven cars act according to the rules of the Nagel–Schreckenberg model, with a few additions to allow for changing lanes. Self-driving cars behave more intelligently, according to rules that are explained in detail below.

Human cars and self-driving cars are distinguished in the model for this paper by a simple set of characteristics that determine how they drive. In this model, human cars randomly slow down with probability 0.1 at each step; self-driving cars do not. Humans change into a lane that has more space ahead with probability 0.25; self-driving cars do this with probability 0.75. Human drivers drive as fast as possible below the specified top speed even if that requires following cars ahead too closely; self driving cars leave a small gap. Human drivers do not check behind when changing lanes to avoid cutting people off; self-driving cars do. Self-driving cars are also assumed in this model to avoid crashes (e.g. using radar, cameras, and a fast computer).

This treatment of human driver behavior is somewhat pessimistic, since some human drivers certainly drive more conscientiously. But some humans also drive more recklessly, or less attentively than this model assumes; there is tremendous variation in human behavior. Accurately describing it would not only require a more complex simulation, but also a large-scale psychological and observational study to determine a distribution of human driver behaviors. Without such a study, a more complex simulation would not be much more informative than the simulation used for this paper.

Despite the apparent pessimism of the model, Nagel and Schreckenberg showed in their original paper that a simple set of similarly pessimistic rules accurately reproduces real traffic flow, a result that has been confirmed in subsequent papers. A simple two-behavior model also has an advantage in explanatory power. It is much easier to explain the effects of varying
proportions of two types of drivers than it is to explain the effects of a distribution of dozens of different behavior types.

The basic structure of the simulation used for this paper is simple:

1) The simulation starts with an empty road vector. Cars are placed randomly on the road with zero speed, and some proportion of the cars is set as self-driving.
2) The speed of each car is updated, according to rules that depend on driver characteristics and the positions of nearby cars.
3) The positions of the cars are updated according to the new speeds. Cell conflicts are resolved either by stopping both cars involved (i.e. a crash), or by minimally altering the cars’ speeds to avoid a crash. Once a car reaches the end of the vector, it goes back to the start and maintains its speed, so the road is effectively a loop.
4) Steps 2 and 3 are repeated many times, with the net flux increased by one each time a car loops around from the end of the road to the beginning.

Each of these steps is explained in detail below.

Step 1: Simulation Initialization

First, the road is initialized as a vector of length 5,000 and width 3. Cars are then placed randomly on it. This random placement is done by calling Matlab’s `randperm` function over the total number of cells, and then setting the positions of the $n$ cars equal to the first $n$ random indices in the array. Over long time scales, the initial positions of cars have no effect on flow.

The number of cars is equal to the specified traffic density times the number of cells on the road. Each space occupied by a car contains a number that acts as the car’s identifier; empty spaces contain zeros. Information about each car is stored in a separate matrix such that the row index corresponds to the number of the car. Each car is either human or self-driving according to the proportion of self-driving cars set for the simulation. In addition to car characteristics, the simulation stores the forward speed, horizontal speed, and position of each car.
Step 2: Calculation of New Speeds

The basic structure for this step comes from the Nagel–Schreckenberg model, with additions made for multiple lanes and more complex car behavior. In the Nagel–Schreckenberg model, new speeds are determined by the following three rules, where the previous speed of the car is \( v \):

1) If \( v \) is less than the speed limit, it is increased by 1.
2) If the next car ahead is within \( v \) cells of the current car, \( v \) is reduced sufficiently so that the current car does not crash.
3) If \( v \) is nonzero, it is reduced by 1 with probability \( p_1 = 0.1 \).

In the model used for this paper, human drivers obey the three rules above, with one addition. With probability \( p_2 = 0.25 \), human drivers change lanes if there is more space ahead in a neighboring lane, and slow down enough to avoid hitting the nearest car ahead in the new lane. Since the positions are all updated in parallel after the new speeds are computed, what they really do is set their horizontal velocity to either 1 or \(-1\), and then change lanes when all the positions are updated.

Self-driving cars follow a slightly different set of rules. Instead of getting as close to the car ahead as possible, they leave a small gap that acts as a buffer so that they do not have to decelerate as rapidly if the car ahead slows down. When changing lanes, they check behind to make sure there are no cars close behind that will be cut off. Self-driving cars also do not slow down randomly.

For comparison, here is the set of rules for self-driving cars:

1) If \( v \) is less than the speed limit, it is increased by 1.
2) With probability \( p_3 = 0.75 \), self-driving cars change lanes (step 2b).
   Otherwise, they do step 2a. For this paper, \( d_1 = 1 \), and \( d_2 = 2 \).
   a) If the next car ahead is fewer than \( v \) sites ahead, \( v \) is reduced sufficiently to avoid a crash. If the car ahead is closer than a preferred distance, \( d_1 \), \( v \) is decreased by one additional unit.
   b) If there is more space ahead and at least a space \( d_2 \) behind in a neighboring lane, the horizontal speed is set to 1 or \(-1\) depending on which lane is available. The forward speed, \( v \), is lowered sufficiently to avoid hitting the next car ahead in the new lane.
Step 3: Updating Positions

The position of each car is updated by moving forward according to the forward speed and changing lanes left or right according to the horizontal speed. The road is effectively a loop, so once a car reaches the end of the vector, it goes back to the start and maintains its speed. If the car’s desired spot is taken, there are two options:

a) The cars crash, so the speeds for both are set to 0 for 60 time steps.
b) The speed and position of the more recently updated car are modified minimally to avoid a crash, and the other car is left alone.

The first option is chosen with probability $p_4$, which was calculated to reflect the actual frequency of crashes on a highway. The Pennsylvania Department of Transportation publishes figures on the total number of miles driven on Pennsylvania interstate highways each year (184.5 hundred million miles in 2010), and the number of crashes on interstate highways each year (9,080 in 2010). These numbers were used to calculate an automobile accident rate of approximately $4.9 \times 10^{-7}$ crashes per mile driven on Pennsylvania interstate highways in 2010 (Pennsylvania 2010). For $p_4=0.0001$, crashes occur with approximately the same frequency in the simulation with only human drivers as in the real world. Traffic flow over long time scales is significantly decreased as the number of crashes increases, so to some extent the effect of self-driving cars on traffic flow depends on how many crashes they can avoid that would otherwise occur with human drivers.

Step 4: Repeat

After updating the positions of the cars, forward speeds are maintained but horizontal speeds are set to 0. Steps 2 and 3 are repeated enough times to minimize uncertainty in results. Every time a car loops from the end of the vector to the beginning, the net flux is increased by 1. For this simulation, 5,000 iterations were used to ensure stable results. The simulation was also repeated for a range of densities, a range of proportions of self-driving cars, and for various values of the parameters $d_1$ and $d_2$. 
## Summary of parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Road size</td>
<td>5,000 cells long, 3 cells wide</td>
</tr>
<tr>
<td>Traffic density</td>
<td>0.05 to 0.85, increments of 0.05</td>
</tr>
<tr>
<td>Fraction of cars self-driving</td>
<td>0, 0.2, 0.5, 0.8</td>
</tr>
<tr>
<td>Random parameters</td>
<td>( p_1 = 0.1, p_2 = 0.25, p_3 = 0.75, p_4 = 0.0001 )</td>
</tr>
<tr>
<td>Preferred follow distance (for self-driving cars)</td>
<td>( d_1 = 1 )</td>
</tr>
<tr>
<td>Distance to check behind before changing lanes (for self-driving cars)</td>
<td>( d_2 = 2 )</td>
</tr>
</tbody>
</table>
VIII. Simulation Results

One goal was to determine the effect of self-driving cars on traffic flow, in particular, to see whether the maximum traffic flow could be increased by adding self-driving cars into the simulation, and whether a high traffic flow could be maintained at higher densities. The simulation showed a significant increase in traffic flow just above and below the critical density, and a smaller but still significant increase in flow at other densities.

A summary of these results is presented in Figure 4, below.

![Figure 4: Primary results from the simulation](image)

The simulation was run with different fractions of self-driving cars: 0 (blue), 0.2 (green), 0.5 (red), and 0.8 (turquoise). The plot is a fundamental diagram with one line for each fraction. The flow is the number of cars that looped from the end to the start of the road over the whole simulation.

Traffic flow is calculated as the number of cars that loop from the end of the road vector to the beginning over the entire time the simulation is run. The absolute numbers are not meaningful without first being translated into real-world time and distance, but comparisons between flow values are meaningful. The highest traffic flow when 80 percent of cars were self-driving was approximately 7,700 cars per 3,600 time steps (averaged over
the course of the simulation) at densities from 0.17 to 0.19, compared to a highest flow of 6,200 cars per 3,600 time steps at densities from 0.19 to 0.25 when no cars were self-driving. This represents an approximately 24 percent increase in peak flow.

When the simulation was run with 80 percent of the cars as self-driving, a flow of at least 6,200 cars per 3,600 time steps was possible up to a density of 0.37: almost twice the density of peak flow without any self-driving cars. In addition, the free flow region extended into higher densities. This is clear from the linear shape of the top line in the density range 0.15 to 0.2, where the other lines — particularly the bottom two — begin to taper off.

The effect of self-driving cars was largest at intermediate densities (just above and below the critical density). At very high densities, the flow with 80 percent of the cars as self-driving approached the flow for the other three simulations. At very low densities, all four simulations gave approximately the same results. One way of understanding this is to look at the variance in car speeds at different densities, shown in Figure 5.

![Figure 5: Variance in car speeds](image)

The variance in car speeds was calculated at every time step and then averaged over all time steps. With more self-driving cars, the variance curve is shifted to the right. This is consistent with the higher critical density observed for the top two lines in Figure 4.
For the plot in Figure 5, the variance in car speeds was calculated at every time step (using the \texttt{var} function in Matlab) and then averaged over all time steps. Note that the speed variance curve for 80 percent of cars self-driving (and to a lesser extent, the curve for 50 percent) is shifted to the right of the curves for 0 percent and 20 percent. This is consistent with the higher critical density observed for the top two lines in Figure 4.

Another goal of this research was to determine optimal values for the parameters $d_1$ (the preferred following distance) and $d_2$ (the distance to check behind when changing lanes) in the model. Sixteen combinations ($d_1, d_2$) were tested, for values from 1 to 4 for each parameter. The results for $d_1=0$ were averaged over all values for $d_2$, likewise for the other values of $d_1$ and the values of $d_2$, and then compared to the results of a simulation with only human drivers. The improvements are shown in Figure 6, below.

![Figure 6: Optimal Values for $d_1$ and $d_2$](image)

The simulation was repeated for sixteen combinations ($d_1, d_2$), for values from 1 to 4 for each parameter. The results for $d_1 = 0$ were averaged over all values for $d_2$, likewise for the other values of $d_1$ and the values of $d_2$, and then compared to the results of a simulation with only human drivers.

The optimal values are $d_1=1$ and $d_2=2$. Higher values of $d_2$ are not so bad, whereas higher values of $d_1$ significantly decrease flow. The optimal value for the combination $(d_1, d_2)$ was $(1, 2)$, although the increase in flow for that combination was within one standard deviation of the flow increases for the combinations $(0,1), (1,1), (0,2), (0,3)$, and $(1,3)$. The standard deviation was calculated by running the human-only simulation 16 times.
IX. Discussion

One surprising result is how close the flow rate is at every density for the simulation with no self-driving cars and the simulation with 20 percent self-driving cars. It appears, at least for this model, that it takes a significant proportion of self-driving cars to make a meaningful difference in traffic flow. In contrast to the findings of Li et al. that a few bad drivers can significantly decrease flow, it seems that it takes a large number of better drivers to significantly increase flow.

Also notable is that Figure 5 agrees with the qualitative description of speed variance given in Section III, even for the simulation with 80 percent of cars self-driving. This suggests that the nature of traffic flow in that case is largely similar to the nature of the flow with no self-driving cars, with one exception. With 50 and 80 percent of cars self-driving, the variance in car speeds did not increase until a higher density. Although turbulence was inevitable in this simulation, self-driving cars were able to delay it by not slowing down randomly, by not over-accelerating as much as human drivers (due to $d_1$), and by not cutting other drivers off (due to $d_2$).

It may be possible to push the critical density even higher by programming self-driving cars more intelligently than was done for this study. There is even more potential for improvement past the critical density, where the speed variance curve with mostly self-driving cars was as high as the curve for only human drivers, just shifted right. It may be possible to decrease speed variance at high densities, which would significantly change the nature of the flow.

Chowdhury et al. argue that there are actually two different types of congested flow: stop-and-go, and synchronized flow (Chowdhury et al. 2000). In synchronized flow, cars move at a uniformly slow speed, but do not regularly come to complete stops. With better programming, it may be possible to create synchronized flow at high densities, instead of stop-and-go traffic. If that is possible, it is also reasonable to expect that the average speed in synchronized flow could be improved, so that flow is increased significantly at high densities, albeit not to the point of free flow.
In addition to studying the results of self-driving cars on highway traffic flow, this simulation also provided an opportunity to study how to best program self-driving cars. Although the model used for this study determines the motion of self-driving cars using a set of rules, Google’s self-driving cars are not actually programmed using a rule-based approach (Sorelle Friedler, personal communication, 2014). Instead, they learn their behavior by tracking skilled human drivers. This ostensibly poses a problem for the basic assumption made in this paper that self-driving cars can be modeled as following a set of rules. However, that problem is no different than the problem of modeling human behavior using a simple set of rules, as Nagel and Schreckenberg originally did. As long as the rule set used to model car behavior reproduces the essential features of that behavior, it is not a problem that Google’s self-driving cars are not actually programmed using a rule-based approach.

The treatment for this simulation was somewhat simplistic in the number of variables taken into account, but it did provide room for experimentation within the range of values for those variables. The two variables studied most closely were following distance and lane-changing behavior.

A common rule of thumb for following distance is that it is smart to stay at least three seconds behind the nearest car ahead when driving at highway speeds. In this simulation, that would mean leaving a gap of cells equal in number to three times the current speed, since each time step corresponds to approximately one second. Human drivers do not always follow this rule perfectly, but self-driving cars could, if it were optimal.

Interestingly, programming the self-driving cars to leave such a large gap decreased traffic flow by at least a factor of two at medium to high densities. Although this rule appears to work in the real world, in this simulation, it clearly does not. Even with a speed of one, a self-driving car will try to leave a gap of three cells, which is far too large for any density higher than approximately 0.33. This is because at a density of 0.33, one third of cells will be occupied, so in general a gap larger than two cells is not possible without coming to a complete stop for one time step.
Upon closer inspection, this rule simply caused self-driving cars to always go slower than was optimal at high densities, which also slowed down cars behind them. This is even taking into account the baseline slow down at high densities. It is possible that a more detailed (but computationally intense) treatment of following behavior could lead to an increase in flow, but it may also be the case that leaving a minimal gap is optimal, provided the car has the resources to brake quickly and avoid rear-end collisions.

A related variable that was not studied was how cars align themselves relative to cars in other lanes, not just the cars immediately ahead in their own lanes. Google’s self-driving cars prefer to stay out of other cars’ blind spots, and adjust their speeds in order to do so (Sorelle Friedler, personal communication, 2014). This makes it less likely for other cars to crash into the self-driving cars. As discussed below, crash frequency has a significant impact on traffic flow. It is possible that changing speed to stay out of another car’s blind spot could increase traffic flow over long time scales, even if it means slowing down more often on a short time scale.

Lane changing behavior was also studied in detail, both for human drivers and self-driving cars. As Li et al. found, aggressive lane changing can have a significant effect on traffic flow at some densities. Any benefit from aggressive lane changing is a result of fast-moving cars not having to slow down as frequently; a downside is that aggressive lane changing can cut off other cars and force them to slow down instead. This is particularly a problem at high densities, when changing lanes almost always leads to cutting in front of another driver.

In this simulation, self-driving cars avoid cutting off other drivers by changing lanes only if there is a gap of at least two cells to the nearest car behind in the new lane. Requiring a larger or smaller gap when changing lanes led to a decrease in flow. This sort of behavior does not actually require a completely self-driving car. Human-driven cars that can detect cars in other lanes could simply flash an alert to their drivers when changing lanes is not optimal, which would have the same effect provided the human drivers listen to the alerts.
Another variable not studied in most papers to date is crash frequency. This is partly because for one- and two-lane roads, the Nagel–Schreckenberg model is automatically collision-free. With only one lane, the only way to crash is by rear-ending a car ahead, which the Nagel–Schreckenberg model prohibits. With two lanes, it is also possible to crash when changing lanes, but this is easy to avoid: cars that want to change lanes just need to check to see if the other lane is open before changing.

With three or more lanes, this becomes much more complicated. For example, two cars in lanes one and three may both want to change into lane two, and both may see that lane two is open. But when the simulation goes to update the car positions, it will run into a conflict even though both cars thought the lane was available. In the real world, this sort of problem is generally avoided using turn signals and horns, although it periodically results in a crash. In the simulation, every addition to behavior requires an additional check for each car in each iteration and can significantly slow down the running time.

A simpler way to avoid crashes is to resolve them by randomly giving one car priority over the other, and placing them with appropriate speeds in the nearest empty spaces. This keeps the running time from being unreasonably slow, and ends up having the same effect as in the real world, where cars resolve conflicts by either staying in their original lanes or slowing down.

Although it sounds nice that the Nagel–Schreckenberg model is crash free, it is unrealistic for just that reason. Crashes do happen, and any accurate model should take that into account, since crashes are a major cause of traffic jams. In the model for this paper, crashes happen with approximately the same frequency as in the real world.

A single crash significantly decreases traffic flow over an extended period of time, so the parameter for crash probability had a large effect mediated through the number of self-driving cars and the frequency with which cars change lanes. Since self-driving cars in this model do not crash and do not cut off other drivers, having them change lanes more frequently increases traffic flow while avoiding the negatives that Li et al. found.
X. Future Research Possibilities

In the near term, the greatest opportunity for improvements to the model is in car following behavior, lane changing behavior, and positioning behavior relative to cars in other lanes. Although a range of parameters was studied for this paper, the code only allowed for a preferred following distance that was the same at every speed. More complex car following behavior could allow self-driving cars to leave large gaps when optimal, and otherwise leave small gaps (e.g. at high densities). This could decrease the variance in car speeds observed at high densities.

Lane changing behavior could also be studied in more detail, especially for roads of three or more lanes. In the current model, self-driving cars check a fixed distance behind in other lanes. Ideally, they would check behind over a larger distance, but still change lanes if a car close behind has a sufficiently slow speed. They might also look multiple cars ahead to see if traffic in one lane is moving generally faster than in another lane, instead of just looking at the distance to the nearest car ahead. This technology could require connected cars, or just self-driving cars with better radar and camera technology.

In the long term, a much wider range of improvements can be imagined. As computers get faster, simulations can become increasingly complex so that they model driver behavior much more accurately. Another area of study completely ignored for this paper is driver behavior as a result of obstacles or unexpected events. The model used for this paper assumes that human drivers always act the same, but that is absolutely not the case. Human behavior is dependent on weather, road conditions, construction, lane blockages, etc. but perhaps most annoyingly, it is also dependent on crashes on the other side of the road. Despite common knowledge that rubbernecking leads to traffic jams, it still happens at every opportunity. In situations like this, self-driving cars could have an even more positive effect on traffic flow — or maybe just get stuck in the jam along with all the human drivers. Only time, and better simulations, will tell.
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XII. References

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